UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRONIC ENGINEERING

MAIN EXAMINATION 2006

TITLE OF PAPER	:	MATHEMATICAL METHODS II (PAPER TWO)
COURSE NUMBER	:	E470(II)
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS. MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

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E470(II) MATHEMATICAL METHODS II (PAPER TWO)

Question one

Given a polynomial function of x as :

$$f(x) = 81\,x^4 - 918\,x^3 + 3537\,x^2 - 5208\,x + 2156$$

(a) plot the given $f(x)$ for $x = 0 \;\; to \;\; 5$. Use *fsolve* command to find its root or roots in the interval of $x = 0 \;\; to \;\; 5$. **(4 marks)**

(b) Transform $f(x) = 0$ into the form $x = g(x)$. Compute a solution of $f(x) = 0$ by Fixed-Point Iteration method, starting from $x_0 = 2$ and doing 5 iterations. Compute the percentage difference of the root found here with the corresponding one obtained in (a). **(7 marks)**

(c) Compute a solution of $f(x) = 0$ by Newton's method, starting from $x_0 = 2$ and doing 5 iterations. Compute the percentage difference of the root found here with the corresponding one obtained in (a). **(6 marks)**

(d) Compute a solution of $f(x) = 0$ by Secant method, starting from $x_0 = 2.0$ and $x_1 = 2.1$ and doing 5 iterations. Compute the percentage difference of the root found here with the corresponding one obtained in (a). **(8 marks)**

Question two

Given $f_0 = f(x_0) = f(1) = 2.18$, $f_1 = f(x_1) = f(2) = 6.23$,

$f_2 = f(x_2) = f(3) = 5.04$, $f_3 = f(x_3) = f(4) = 3.35$ and

$f_4 = f(x_4) = f(5) = 4.76$,

(a) (i) use the Newton's divided difference interpolation formula , i.e.,

$$f(x) \approx f_0 + (x - x_0)f[x_0,x_1] + (x - x_0)(x - x_1)f[x_0,x_1,x_2] + \dots ,$$

find the values of $f[x_0,x_1]$, $f[x_0,x_1,x_2]$, $f[x_0,x_1,x_2,x_3]$ and

$f\left[x_0,x_1,x_2,x_3,x_4\right]$ and thus the interpolated $f(x)$. Find the value of

$f(2.4)$. **(10 marks)**

(ii) use the Lagrange interpolation , i.e.,

$$P_4(x) = f_0 \frac{(x - x_1)(x - x_2)(x - x_3)(x - x_4)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4)} + \cdots\cdots\cdots ,$$

find $P_4(x)$. Find the value of $P_4(2.4)$. **(7 marks)**

(b) Set $f(x) \approx k_0 + k_1 x + k_2 x^2 + k_3 x^3$ and use the method of the least squares to find the values of k_0 , k_1 , k_2 *and* k_3 . Find the value of $f(2.4)$. Compare it with that obtained in (a)(i) and find their percentage difference.(**8 marks**)

Question three

(a) Given the following definite integral

$$\int_0^6 e^{-\sqrt{x}}\left(15 + 98\,x - 91\,x^2 + 25\,x^3 - 2\,x^4\right)dx$$

(i) plot the integrand for $x = 0$ *to* 6 **(2 marks)**

(ii) use int command to find the value of the given integral, **(2 marks)**

(iii) divide the integration range into ten equal intervals and compute the value of the given integral by the trapezoidal rule. Compare this value with that obtained in (ii) and evaluate their percentage difference. **(5 marks)**

(iv) divide the integration range into ten equal intervals and compute the value of the given integral by Simpson's rule. Compare this value with that obtained in (ii) and evaluate their percentage difference. Make a brief remark on the accuracy of this method compared to that of the method in (iii) . **(5 marks)**

(b) Given the following system of linear equations :

$$\begin{cases} 2\,x_1 - 17\,x_2 - 3\,x_3 = 34 \\ 13\,x_1 + 4\,x_2 + 5\,x_3 = 29 \\ x_1 + 2\,x_2 - 20\,x_3 = -11 \end{cases}$$

(i) use *solve* command to find the solutions of x_1, x_2 *and* x_3 for the given system of linear equations , **(2 marks)**

(ii) apply the Gauss-Seidel iteration (5 steps) to the given system, starting from ($x_1 = 1$, $x_2 = 1$ and $x_3 = 1$). Compute the iterated solutions of the system. Compare the iterated solutions with the solutions obtained in (b)(i) and compute their respective percentage differences . **(9 marks)**

Question four

(a) Given the following differential equation $\dfrac{d\,y(x)}{d\,x} = (x+1)^2 - y(x)$ and initial condition $y(0) = 3$,

(i) use $dsolve$ command to find its specific solution of $y(x)$. Evaluate the value of $y(0.8)$. **(2 marks)**

(ii) use Euler method by starting with $x = 0$ and $h = 0.1$, do 8 steps to find the approximate value of $y(0.8)$. Compare it with that obtained in (a)(i) and estimate their percentage difference. **(5 marks)**

(iii) use Runge-Kutta method by starting with $x = 0$ and $h = 0.1$, do 8 steps to find the approximate value of $y(0.8)$. Compare it with that obtained in (a)(i) and estimate their percentage difference. **(6 marks)**

(b) Given the following function of x and y as :

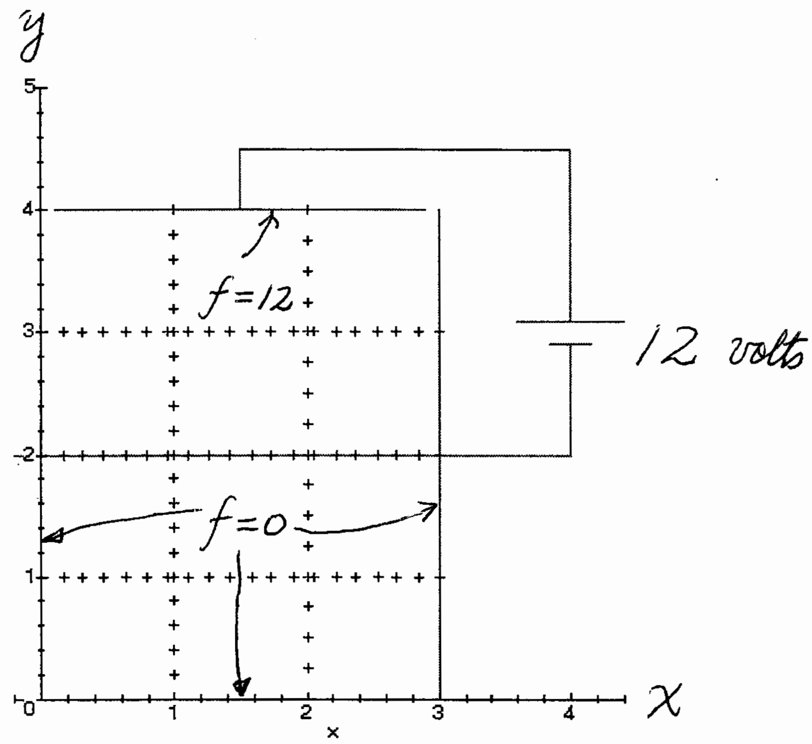
$$f(x,y) = 8\,x^2 - 20\,x\,y + 17\,y^2 - 32\,x + 40\,y \quad ,$$

(i) find the extremum value of f and the position of (x,y) that the extremum happens by solving $\dfrac{\partial f}{\partial x} = 0 \quad and \quad \dfrac{\partial f}{\partial y} = 0$, **(2 marks)**

(ii) use the method of steepest descent , starting with the point $(x_0 = 0 , y_0 = 0)$, do 3 steps to find the approximate extremum value of f . Compare it with that obtained in (i) and estimate their percentage difference. **(10 marks)**

Question five

An infinite long, rectangular U shaped conducting channel is insulated at the corners from a conducting plate forming the fourth side with interior dimensions as shown below :



Given the boundary conditions as $f(0,y)=0$, $f(3,y)=0$, $f(x,0)=0$ and $f(x,4)=12$ *volts* ,

(a) use the discrete Laplace equation , i.e.,

$$f(i,j)=\frac{f(i-1,j)+f(i+1,j)+f(i,j-1)+f(i,j+1)}{4}$$

where $i=1,2$ and $j=1,2,3$, to find the values of $f(x,y)$ at the six mesh points , i.e., the values of $f(1,1)$, $f(1,2)$, $f(1,3)$, $f(2,1)$, $f(2,2)$ and $f(2,3)$, (10 marks)

Question five (continued)

(b) assign the values of $f(1,1)$, $f(1,2)$, $f(1,3)$, $f(2,1)$, $f(2,2)$ and $f(2,3)$ all as 1 , use the renumerating scheme and do 5 rounds of renumerating to find their renewed values . Compare the final renewed value of $f(1,1)$ with that obtained in (a) and estimate their percentage difference. (15 marks)